1984: PART A

1) $f(x) = x^3 - 7x + 6$

a) $x^3 - 7x + 6 = 0$

1	1	0	-7	6
		1	1	-6
	1	1	-6	0

$(x-1)(x^2+x-6) = 0$
$(x-1)(x-2)(x+3) = 0$
$x = 1, x = 2, x = -3$

b) $f'(x) = 3x^2 - 7$
$f'(-1) = 3(1) - 7 = -4$

$f(-1) = (-1)^3 - 7(-1) + 6 = 12$

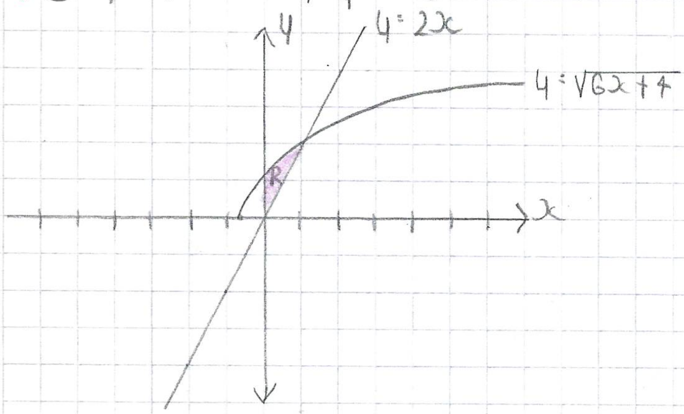
Equ. of tangent:
$y - 12 = -4(x+1)$
$y - 12 = -4x - 4$
$y = -4x + 8$

c) $f'(c) = \frac{f(3) - f(1)}{3-1} = 3c^2 - 7$

$3c^2 - 7 = \frac{(3^3 - 21 + 6) - (1 - 7 + 6)}{2}$

$3c^2 - 7 = 6$
$3c^2 = 13$
$c^2 = \frac{13}{3}$
$c = \sqrt{\frac{13}{3}}$

2) $y = \sqrt{6x+4}$, $y = 2x$



Intersection: $2x = \sqrt{6x+4}$
$4x^2 = 6x + 4$
$2x^2 - 3x - 2 = 0$
$2x^2 + x - 4x - 2 = 0$
$x(2x+1) - 2(2x+1) = 0$
$(x-2)(2x+1) = 0$
$x = 2, x \neq -\frac{1}{2}$

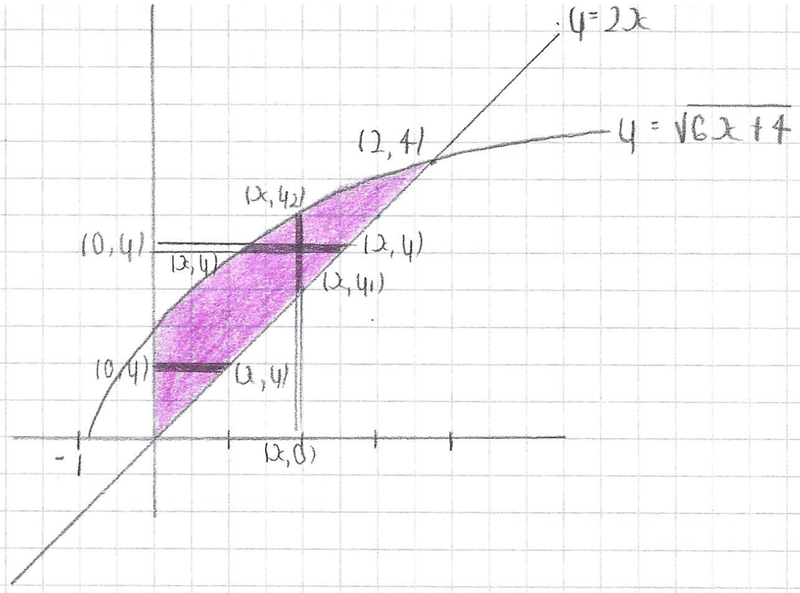
a) Area $R = \int_0^2 (\sqrt{6x+4} - 2x)\,dx$

$= \left[\frac{1}{9}(6x+4)^{3/2} - x^2\right]_0^2$

$= \frac{1}{9}(16)^{3/2} - 4 - \frac{1}{9}(4)^{3/2}$

$= \frac{1}{9}(64) - 4 - \frac{1}{9}(8) = \frac{20}{9}$

b)



$R(x) = y_2 - 0 = \sqrt{6x+4}$
$r(x) = y_1 - 0 = 2x$

$$\text{Volume} = \pi \int_0^2 [(\sqrt{6x+4})^2 - (2x)^2]\,dx$$
$$= \pi \int_0^2 (6x+4-4x^2)\,dx$$
$$= \frac{28}{3}\pi$$

c) $R(x) = x - 0 = \frac{y}{2}$ $\qquad y = 2x,\quad x = \frac{y}{2}$

$R(x) = x - 0$ $\qquad y = \sqrt{6x+4}$
$r(x) = x - 0 = \frac{y^2-4}{6}$ $\qquad y^2 = 6x+4,\quad x = \frac{y^2-4}{6}$

$$\text{Volume} = \pi \int_0^2 \left(\frac{y}{2}\right)^2 dy + \pi \int_2^4 \left(\frac{y}{2}\right)^2 - \left(\frac{y^2-4}{6}\right)^2 dy$$
$$= \frac{416}{135}\pi$$

189: PART B

4) $f(x) = \dfrac{x}{\sqrt{x^2-4}}$

a) Domain: $x^2 - 4 > 0$
$x^2 > 4$
$x > \pm 2$

b) vertical asymptote when undefined:
$x^2 - 4 = 0$
$x^2 = 4$
$x = \pm 2$

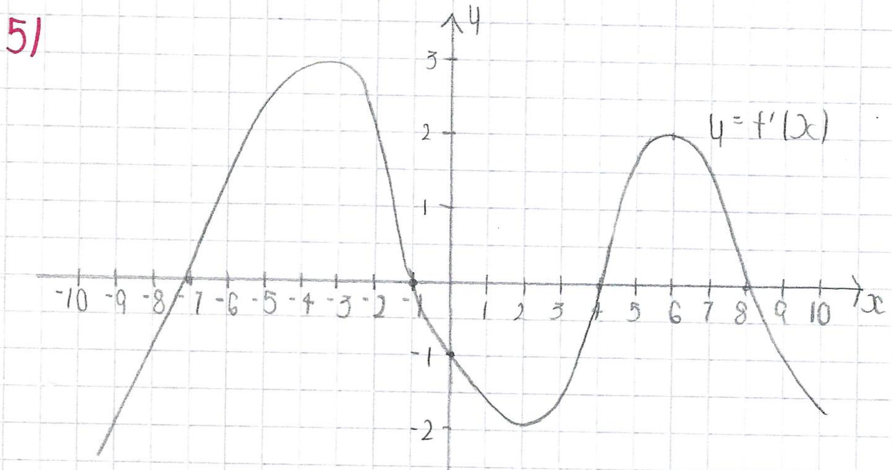
$x = -2, x = 2$

c) Horizontal asymptote:
$$\lim_{x\to\infty} \frac{x}{\sqrt{x^2-4}} = \lim_{x\to\infty}\left(\frac{x/x}{\sqrt{\frac{x^2-4}{x^2}}}\right) = \lim_{x\to\infty}\left(\frac{1}{\sqrt{1-\frac{4}{x^2}}}\right) = \frac{1}{\sqrt{1-0}} = 1$$
$$\lim_{x\to-\infty} \frac{x}{\sqrt{x^2-4}} = \lim_{x\to-\infty}\left(\frac{1}{-\sqrt{1-\frac{4}{x^2}}}\right) = -\frac{1}{\sqrt{1-0}} = -1$$

$y = -1, y = 1$

d) $f(x) = \dfrac{x}{(x^2-4)^{1/2}}$
$$f'(x) = \frac{(x^2-4)^{1/2}(1) - x(\tfrac{1}{2}(x^2-4)^{-1/2})(2x))}{(x^2-4)^{1/4}}$$
$$= \frac{(x^2-4)^{1/2} - x^2(x^2-4)^{-1/2}}{(x^2-4)^{1/4}}$$
$$= \frac{(x^2-4)^{-1/2}[(x^2-4) - x^2]}{(x^2-4)^{1/4}}$$
$$= \frac{-4}{(x^2-4)^{3/4}}$$

5)



a) horizontal tangent has slope of 0
$x = -7, x = -1, x = 4, x = 8$

b) Rel. max when $x = -1, x = 8$ since sign of $f'(x)$ changes from +ve to −ve at $x = -1$ and $x = 8$

b)

c) concave downward on $(-2, 2)$ and $(6, 10)$ since $f'(x)$ is
on those intervals

6)

c)